Enrollment No: _______________________ Exam Seat No:_______________________

# C.U.SHAH UNIVERSITY

## Winter Examination-2015

**Subject Name : Linear Algebra-I**

**Subject Code : 4SC03MTC2** **Branch :B.SC (Mathematics,Physics)**

**Semester :3 Date :5/12/2015 Time :2:30 To 5:30 Marks :70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1** **Attempt the following questions:** **(14)**

**a)** True/false: Union of two subspaces is also subspace.

**b)** What are the standard basis of $R^4$?

**c)** If $V=R^+$ and x + y = x . y , k x = $x^k$ is vector space then write the zero element of V.

**d)** Write the matrix of linear transformation which is responsible for reflection with respect to y axis.

**e)** What is inner product Space?

**f)** True/false: $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is the matrix of linear transformation which is responsible for rotation by an angle $\theta$.

**g)** Define norm of vector in inner product space .

**h)** Are $(8, \frac{2}{3}, -1)$ and $(-4, -\frac{1}{3}, \frac{1}{2})$ are linearly dependent ? Justify your answer.

**i)** Write the basis of $\mathcal{P}_2$ .

**j)** What is dimension of vector space?

**k)** True/false: if W is subspace of finite dimensional vector space then dimW $\leq$ dim V .

**l)** Find the angle between (1,2,0) and (-2,1,5) .

**m)** Define kernel of linear transformation.

**n)** What is span of (1,0) and (0,1)?

**Attempt any four questions from Q-2 to Q-8**

**Q-2** **Attempt all questions** **(14)**

**a)** Which of the following are subspace of V . **(8)**

(1) W=(a,b,c) / a$\geq$ 0 } $V=R^3$ .

(2) W=(a,b,c) / a$= 0$ } V=$R^3$ .
(3) W=(a,b,c) / a$b = 0$ } V= $R^3$ .
(4) W=(a,b,c) / $\sqrt{3}a = \sqrt{5}b$ } V= $R^3$

**b)** Define vector space and show that $R^n$ is a vector space . **(6)**

**Q-3** **Attempt all questions** **(14)**

**a)** If V is vector space and $W_1$ , $W_2$ are two subspace of V then show that $W_1 \cap W_2$ and $W_1 + W_2$ is also subspace of V. **(8)**

**b)** Define subspace of vector space.Let V is vector space W⊂ V.then show that W is subspace of V if and only if α u+ $\beta v$ ∈ W for all $\alpha$ , $\beta$ ∈ R and u,v ∈ W . **(6)**

**Q-4** **Attempt all questions** **(14)**

**a)** Examine the sub sets of $R^3$ are L.D... or L.I. **(6)**
(1) { ( 1,2,1) ,(-1,1,0) ,(5,-1,2)}
(2) {(1,2,1) ,(-1,1,0)}

**b)** Check whether (1,2,4) ,(1,5,4) ,(0,1,2) ∈ span A **(6)**
Where A={ (0,1,-1) ,(0,0,2) ,(1,3,0) } .

**c)** Define span of {u,v} **(2)**

**Q-5** **Attempt all questions** **(14)**

**a)** Check which of the following are L.T.? **(6)**
(a) T: $R^2 \rightarrow R^3$ , T(x,y)=(x,xy,$x^2$y)
(b) T: $R^3 \rightarrow R^3$ , T(x,y,z)=(x+y,y+z,z-y)

**b)** Prove that for any two vectors x ,y ∈ V $\|x+y\|^2+\|x-y\|^2=2(\|x\|^2+\|y\|^2)$ . **(4)**

**c)** Show that V=C[0,1] with $\langle f,g\rangle=\int_0^1 f(t)g(t)dt$ is an inner product space. **(4)**

**Q-6** **Attempt all questions** **(14)**

**a)** State and prove rank -nullity theorem. **(8)**

**b)** Examine the sub sets of C[0,2$\pi$] are L.D... or L.I. **(6)**
(1) { sinx , cosx , $e^x$ }
(2) { x ,$x^2$ ,$x^3$ }

**Q-7** **Attempt all questions** **(14)**

**a)** Verify rank nullity theorem for T:$R^4 \rightarrow R^2$ such that **(6)**
$T(x_1,x_2,x_3,x_4)=(x_1-x_2+x_3-x_4, 2x_1+x_2+3x_3+x_4)$

**b)** If T:V→W is linear transformation. Then show that **(6)**
(a) T(0) = 0
(b) T(-u) = -T(u)
(c) T(u-v) = T(u)-T(v)

**c)** Define direct sum of two subspaces. **(2)**

**Q-8** **Attempt all questions** **(14)**

**a)** Prove that {(1,2,1),(2,1,0),(1,-1,2)} forms a basis of $R^3$. **(6)**

**b)** If v,$v_1$, $v_2$ ,…… $v_n$ are vectors of vector space V .v is linear combination of $v_1$, $v_2$ ,…… $v_n$ then show that { v,$v_1$, $v_2$ ,…… $v_n$} is L.D. **(6)**

**c)** State Pythagorean theorem in inner product space. **(2)**